The Mathematics of Peter Lowe's "Spiral" Works

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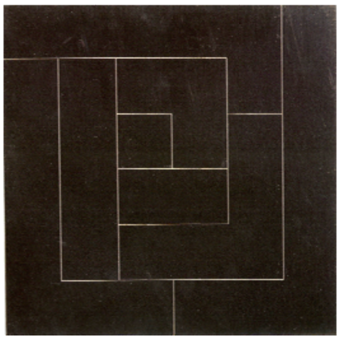
Figure 1 - Spiral of 8 Integers (1963)



Introduction

Peter Lowe and I met at Goldsmiths' College, London in 1979 whilst I was still an undergraduate. He showed me one of his earlier works entitled "Spiral of 8 Integers" (1963) and asked me the question "what are the next few squares in the sequence".

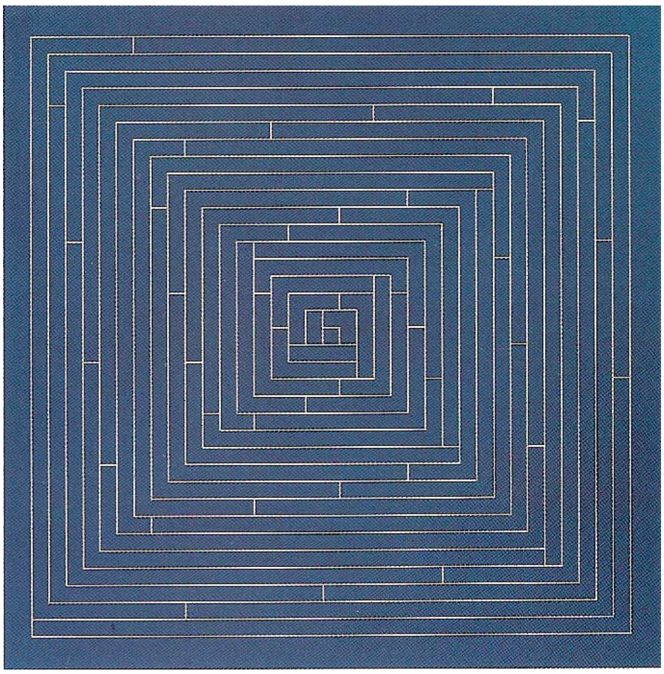
I saw Peter's problem in terms of a finite arithmetic series (for example, 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) whose sum must equal the square of the spiral work (in this example, 36).

Sequences and Series

One of my maths lecturers joked "only a *mathematician* knows the difference between a sequence and a series". Perhaps he was right.

A *sequence* of numbers is a comma-separated list of numbers, each having a specific relationship to its predecessor. The Triangular Number sequence is a case in point: 1, 3, 6, 10, 15, etc.

Figure 2 - Spiral Without Border



A *series* is the sum of a sequence of numbers. For example, the Geometric series: 1 + ½ + ¼ + 1/8 + etc.

To sum a finite arithmetic series with maximum value n (8 in this example) the following general formula is applied:

```
Sum(n)  = n(n+1)/2
```

In our example:

```
Sum(8)   = 8(8+1))/2
         = 36.
```

We want this value to be the square of an integer m. Where m is 6 and the square of m is 36 then the condition we seek is satsfied and derive the first "spiral" work.

Mathematically speaking, we are solving the following equation:

$$m^2 = [n(n+1)]/2$$

and to determine n in terms of m we must satisfy the following equation:

$$n = [\sqrt{(1+8m^2)} - 1]/2 \qquad (1)$$

Computing a Solution

The Spiral problem can be solved using a computer with an algorithm to output values for n when the following conditions apply:

n satisfies equation (1) **and** n is an integer with a decimal fraction = 0.

Although personal computers had emerged by 1979, they were expensive and beyond the means of my student budget. Instead, I had a Texas Instruments TI 57 programmable calculator which I used to address the spiral problem.

I wrote several programs to iterate for n and to test that n was an integer. Appendix 1 shows a program I wrote in 1979 relating back to the "spiral" problem. I eventually calculated the first five or six values before falling foul of the calculator's inherent limitations of speed and accuracy. The following table show these first few values:

m	n
1	1
8	6
49	35
288	204
1681	1189
9800	6930
57121	40391

The Spiral Factor

Not only was the sequence of "spiral" numbers a novelty (to me) at that time but I also noticed the ratio between any number and its predecessor appeared to converge to the irrational value $3+2\sqrt{2}$ – a value of 5.8284271 when truncated to seven decimal places. I call this convergence factor the ***spiral factor***.

The program in **Appendix 1** used the *spiral factor* to approximate the next value of **m** by multiplying the current value of **n** by the *spiral factor*. The more accurate this factor became, the nearer one got to the next value of **m**, thereby dramatically reducing the number of iterations required to compute the next value of **n**.

An Updated Program

Since the Texas Instruments TI 57 programmable calculator is no longer manufactured, an updated version of the program in **Appendix 1** was written in the UNIX **bc** utilty language. **Appendix 2** shows the code and **Appendix 3** lists the initial 51 and final 3 results of the 3924 values calculated by the

algorithm. The *spiral factor* helped to reduce the calculation time of these values to only one hour on a typical laptop computer.

Conclusion

Rather than starting from the mathematics of triangular and square numbers and deriving a work of art, this approach reversed the process. Given a "Spiral" by Peter Lowe dating back to 1963, I computed an algorithm in 1979 - which took advantage of a *spiral factor* to accelerate the speed of convergence – to determine the next spiral works in the sequence.

Although Sengupta [1] used MATHEMATICA in a similar approach, no use of the *spiral factor* is mentioned in his paper or in similar investigations of square and triangular numbers [2][3].

References

1. Sengupta, D. C, (1999). *Triangular Squares using MATHEMATICA*, Proceedings of the Fourth Asian Technology Conference in Mathematics, Guangzhou, China.
2. Peters, H. and Schoenmakers, G. (2002). *Mathematical Modelling in Maastricht*. University of Maastricht. 8th MMM – Problem 5.
3. Dudeney, H. E. (1917). *Amusements in Mathematics*. Project Gutenberg. Section 137, A Study In Thrift – solution.

Appendix 1

A Texas Instrument s TI 57 programmable calculator (LED version) program relating to the “spiral” problem:

TITLE
TITEL
TITRE

PROGRAMMER
PROGRAMMIERER
PROGRAMMEUR
E Grabecki

DATE
DATUM
DATE
14-1-1979

TI PROGRAMMABLE 57
PROGRAM RECORD
PROGRAMM-BERICHT
FICHE PROGRAMME

PROGRAM DESCRIPTION • PROGRAMM BESCHREIBUNG • DESCRIPTION DU PROGRAMME

The Arithmetic Square problem.

USER INSTRUCTIONS • BENUTZER INSTRUKTIONEN • MODE D'EMPLOI

STEP SCHRITT SEQ.	PRESS BEFEHL APPUYER SUR	DISPLAY ANZEIGE AFFICHAGE
	STO 0	5.8284271 *
	STO 1	1
	STO 2	0
	STO 7	0.5
	press LRN and	Key-in the program.
	.... press LRN	again, and then RST,
	now press	R/S ....
		.... first № will display.
	* (17-3-79) does 5.8284271....	= $(3+2\sqrt{2})$?

TEXAS INSTRUMENTS

001

FLOW CHART/NOTES
FLUSSDIAGRAMM/BEMERKUNGEN
ORGANIGRAMME/NOTES

The display will stop – after pressing R/S – at each main value of the number sequence, and only pause at the last term of the arithmetic progression which when totalled would give the square of the previous number displayed

KEY / TASTE / TOUCHE	LOC / ADR / ADR	CODE / KODE / CODE	COMMENTS / BEMERKUNGEN / COMMENTAIRES
2nd LBL 1	00		
RCL 1	01		
×	02		
RCL 0	03		
=	04		
STO 2	05		
INV 2nd Int	06		
2nd x≷t	07		
GTO 2	08		
RCL 2	09		
2nd Int	10		
STO 1	11		
R/S	12		
GTO 3	13		
2nd LBL 2	14		
RCL 2	15		
2nd Int	16		
+	17		
1	18		
=	19		
STO 1	20		
R/S	21		
GTO 3	22		
2nd LBL 3	23		
RCL 1	24		
x^2	25		
×	26		
8	27		
+	28		
1	29		
=	30		
$\sqrt{x}$	31		
÷	32		
2	33		
−	34		
1	35		
=	36		
2nd x≷t	37		
GTO 4	38		
2nd Int	39		
2nd PAUSE	40		
RST	41		
2nd LBL 4	42		
2nd Int	43		
+	44		
1	45		
=	46		
2nd PAUSE	47		
RST	48		
	49		

DATA REGISTERS / DATENSPEICHER / REGISTRES-MEMOIRE

			LABELS / LABELS / LABELS	
0	Dsz	n	0	
1		Σy	1	
2		Σy^2	2	
3		Σx	3	
4		Σx^2	4	
5	(AOS)	Σxy	5	
6	(AOS)		6	
7	(t)	i + 1	7	
			8	
			9	

TEXAS INSTRUMENTS

Appendix 2

bc12.txt is a UNIX **bc** utility script. To run it, type the following command in a Bourne shell:

```
$ bc bc12.txt >eddytemp12
```

A partial listing of the output is given in **Appendix 3**.

```
define as(x) {
 return((sqrt(1+8*x*x)-1)/2);
}

scale=3000
c=3+(2*sqrt(2))
m=0
n=0
r=0

for (p=1; p > 0; p++) {

   scale=0
   m=p/1

   scale=3000
   n=as(m)

   scale=0
   nq=n/1
   nr=n%1

   if (nr == 0) {
      r=r+1
      print "Result: ", r,  "\n";
      print "m = ",  m,   "\n";
      print "n = ",  nq, "\n\n";
      if (p > 49) p = m * c;
   }

}
```

Appendix 3

The following is a partial listing of the UNIX bc script run in **Appendix 2**. The script was run on a Dell Latitude D620 laptop computer and generated 3924 sequence values in one hour. Only the first 51 and the final three results are listed here:

```
Result: 1
m = 1
n = 1

Result: 2
m = 6
n = 8

Result: 3
m = 35
n = 49

Result: 4
m = 204
n = 288

Result: 5
m = 1189
n = 1681

Result: 6
m = 6930
n = 9800

Result: 7
m = 40391
n = 57121

Result: 8
m = 235416
n = 332928

Result: 9
m = 1372105
n = 1940449

Result: 10
m = 7997214
n = 11309768

Result: 11
m = 46611179
n = 65918161

Result: 12
m = 271669860
n = 384199200

Result: 13
m = 1583407981
n = 2239277041

Result: 14
m = 9228778026
n = 13051463048

Result: 15
m = 53789260175
n = 76069501249
```

```
Result: 16
m = 313506783024
n = 443365544448

Result: 17
m = 1827251437969
n = 2584123765441

Result: 18
m = 10650001844790
n = 15061377048200

Result: 19
m = 62072759630771
n = 87784138523761

Result: 20
m = 361786555939836
n = 511643454094368

Result: 21
m = 2108646576008245
n = 2982076586042449

Result: 22
m = 12290092900109634
n = 17380816062160328

Result: 23
m = 71631910824649559
n = 101302819786919521

Result: 24
m = 417501372047787720
n = 590436102659356800

Result: 25
m = 2433376321462076761
n = 3441313796169221281

Result: 26
m = 14182756556724672846
n = 20057446674355970888

Result: 27
m = 82663163018885960315
n = 116903366249966604049

Result: 28
m = 481796221556591089044
n = 681362750825443653408

Result: 29
m = 2808114166320660573949
n = 3971273138702695316401

Result: 30
m = 16366888776367372354650
n = 23146276081390728245000

Result: 31
m = 95393218491883573553951
n = 134906383349641674153601

Result: 32
m = 555992422174934068969056
n = 786292024016459316676608

Result: 33
```

```
m = 3240561314557720840260385
n = 4582845760749114225906049

Result: 34
m = 18887375465171390972593254
n = 26710782540478226038759688

Result: 35
m = 110083691476470624995299139
n = 155681849482120242006652081

Result: 36
m = 641614773393652358999201580
n = 907380314352243226001152800

Result: 37
m = 3739604948885443528999910341
n = 5288600036631339114000264721

Result: 38
m = 21796014919919008815000260466
n = 30824219905435791458000435528

Result: 39
m = 127036484570628609361001652455
n = 179656719395983409634002348449

Result: 40
m = 740422892503852647351009654264
n = 1047116096470464666346013655168

Result: 41
m = 4315500870452487274745056273129
n = 6103039859426804588442079582561

Result: 42
m = 25152582330211071001119327984510
n = 35571123060090362864306463840200

Result: 43
m = 146599993110813938731970911633931
n = 207323698501115372597396703458641

Result: 44
m = 854447376334672561390706141819076
n = 1208371067946601872720073756911648

Result: 45
m = 4980084264897221429612265939280525
n = 7042902709178495863723045838011249

Result: 46
m = 29026058213048656016282889493864074
n = 41049045187124373309618201271155848

Result: 47
m = 169176265013394714668085071023903919
n = 239251368413567743993986161788923841

Result: 48
m = 986031531867319631992227536649559440
n = 1394459165294282090654298769462387200

Result: 49
m = 5747012926190523077285280148873452721
n = 8127503623352124799931806454985399361

Result: 50
m = 33496046025275818831719453356591156886
```

```
n = 4737056257481846670893653996045000896 8

Result: 51
m = 1952292632254643899130314399906734885 95
n = 2760958718255586754536874333077146544 49

…

Result: 3922
m = 549415365334901729251713051141838537062799585779102975495481655 7\
0826239033138522890069641453925188930163636286855364897881538275639 0\
5122952962510358840738388240600410553286372130950903601172909428138 6\
0042460166960310789448731816956241910093284670587917328357998116899 5\
5545543943711086325156781659911784021424037250065254952798671268057 9\
4464870580557486430049356147195968230125334900275650263218637828610 3\
4159066041363003782931285752903779660767824153942599658107311893978 0\
9169467961134821500481593491231939229110608723875551894704608903949 1\
7255770114774441300808594941769508450462614656217234799879569246442 8\
2727482033858050436462240084310091095305302538020770668548399626807 4\
6806614532220173792095354664096667415156829427244529641527372486070 8\
4423611000561487289140069514560860032173661341228438421343939308486 2\
1368367019038146155586937664490277811825250947310136035845696836898 3\
3697824138405564336556074775674652340070369731306109987083552683010 4\
9601077869702605768761552222683762554368504328281724007120642499943 2\
6689322475598096903541364476172234084615867681605480330757776453833 6\
1483159185288706745510777189744897773714738900126301177149978861687 7\
4926587413390918033581000173149751374687208038145895654212835883854 9\
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8156654060188096362629017320656133531583445954351476236053294760434 5\
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8581218484664759908527546848425892134178805023353056298570615064898 6\
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3645921854508776080019414761114819948488400552595927641137231171108 5\
6984971474171333255412134213274578543618381335539761769693336414834 8\
0999125307093481143991355126189052624168273085144332211022455205167 3\
8657161613576853246296654794673635581567784178013623295573147807964 3\
4008569125623592634006082371491280988595730644885995116366181369258 5\
6576645743851514282148319617356309694441721518015796767020362903552 3\
6832189800497229091225503477012535369811253707406183955721441419663 1\
2413072623161601084684831983216896753813720382137863008396299962069 9\
00687509766134
n = 776990661032786849987047508704181076063024034948737173814978613 7\
5084340668856386369079386643270915274167974076965003726989371182444 5\
5055231646825446778302411824488122056155777599114463639898060123297 9\
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```

```
4513674786947195012039890780555818585800495518153683916695958597978 3\
```

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2073222112966420312847211611728550819732485861838771443002414065822 0\
0244878856939294171147856176363298985224973574166582598666623588652 9\
6158734461857801131236261649439126974306852466252099968681579626926 1\
1006168286548873796660221766964445259775215087701570083304279960394 4\
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0464305956856945188118671877740274780512742730858778443961224570514 9\
2626416686702537807874773972539625749676294594593815757745623870513 1\
8034322181063684346022374124650045929043905341297790947591978360452 6\
4815686723995214679473520880959706641437010696122111899227763461196 0\
2369748310795138476808669500184554032760965880217239932041081649462 8\
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3690359852250724461591656128994961818981905311289430046901886411885 5\
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2113936436227414879853729742513454567057886113594645745888626109554 0\
7099702936928966565610017566614794019708649790838159673748912981876 8\
2459703238617868293627691306318885940423559727302287001827097110992 4\
1370013082555679903029356002456594349853306204764773836128615122047 9\
6552278299053966400893652123888696176480804051751885074146109385718 7\
4335491770634574820520645683691003660778586525022831455908284345505 2\
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4754172232250298792083287561904758739374745745851066327827652084659 1\
1664963043869479145669864248791036046901870707055746279561632494495 4\
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5048929785999330121358158130293324692926718215707204611350821551702 6\
2615644053848638613129388843479315902217898061200751244553030129721 7\
6840462249909922527463485037677601016743236140812403476400794415442 8\
830356300081521

Result: 3924
m = 1866394914308677159712369197386142804282388137762959666920293151\
2050581515896315255601365444359791951027975058001212479878586048751 9\
8477529801245814616321815419840644440797376574155389248987161817579 3\
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